

# Large $\tan \beta$ effects in flavour physics

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**Abstract.** After a short introduction to the SUSY flavour problem, we focus the attention on the MSSM with MFV and large  $\tan \beta$ . The theoretical motivations and the general features of this scenario are briefly reviewed. The possible signatures in low-energy flavour-violating observables are discussed, with particular attention to the role played by  $\mathcal{B}(P \rightarrow \ell \nu)$ ,  $\mathcal{B}(B_{s,d} \rightarrow \ell^+ \ell^-)$  and  $\mathcal{B}(\mu \rightarrow e \gamma)$ .

**PACS.** 12.60.Jv Supersymmetric models

## 1 Introduction: the SUSY flavour problem and the MFV hypothesis

In most extensions of the Standard Model (SM), including the so-called MSSM (Minimal Supersymmetric SM), the new degrees of freedom which modify the ultraviolet behavior of the theory appears only around or above the electroweak scale ( $v \approx 174$  GeV). As long as we are interested in processes occurring below this scale (such as  $B$ ,  $D$  and  $K$  decays), we can integrate out the new degrees of freedom and describe the new-physics effects –in full generality– by means of an Effective Field Theory (EFT) approach. The SM Lagrangian becomes the renormalizable part of a more general local Lagrangian which includes an infinite tower of higher-dimensional operators constructed in terms of SM fields. The higher-dimensional operators are suppressed by inverse powers of a dimensional parameter, the effective scale of new physics, which in the MSSM case can be identified with the mass scale of the soft-breaking terms.

This approach allows us to analyse all realistic extensions of the SM in terms of a limited number of parameters. In particular, it allows us to investigate the flavour-symmetry breaking pattern of the model without knowing the dynamical details of the theory above the electroweak scale. In case of a generic flavour structure, the higher-dimensional operators should naturally induce large effects in processes which are not mediated by tree-level SM amplitudes, such as  $\Delta F = 1$  and  $\Delta F = 2$  flavour-changing neutral current (FCNC) transitions. Up to now there is no evidence of these effects and this implies severe bounds on the effective scale of new physics. For instance the good agreement between SM expectations and experimental data on  $K^0 - \bar{K}^0$ ,  $B_d - \bar{B}_d$ , and  $B_s - \bar{B}_s$  mixing amplitudes leads to bounds above  $10^4$  TeV,  $10^3$  TeV, and  $10^2$  TeV, respectively. In the MSSM, where the new degrees of

freedom are expected to be around the TeV scale, these bounds represent a serious problem: if we insist that squarks in the TeV range are necessary for a stabilization of the Higgs sector, we have to conclude that the model has a highly non-generic flavour structure, similar to the SM one.

The quark-flavour structure of the SM is quite specific. The gauge sector is invariant under a large global symmetry,

$$SU(3)_{Q_L} \times SU(3)_{D_R} \times SU(3)_{U_R} , \quad (1)$$

corresponding to the family mixing of the three independent fermion fields. This symmetry is broken only, and in a well-defined way, by the Yukawa interaction. The two Yukawa couplings  $Y_U$  and  $Y_D$  introduce breaking terms of the type

$$Y_U \sim 3_{Q_L} \times \bar{3}_{U_R} , \quad Y_D \sim 3_{Q_L} \times \bar{3}_{D_R} , \quad (2)$$

which are highly hierarchical (with large entries only for the third family) and quasi aligned in the  $SU(3)_{Q_L}$  sub-space (with the misalignment controlled by the off-diagonal entries of the CKM matrix). This specific symmetry and symmetry-breaking structure is responsible for the successful SM predictions in the quark-flavour sector.

A natural and consistent possibility to export this pattern in the MSSM (as well as in other TeV-scale new-physics scenarios) is what goes under the name of Minimal Flavour Violation (MFV) hypothesis [1]. According to this hypothesis,  $Y_U$  and  $Y_D$  are the only breaking sources of the flavour symmetry also beyond the SM. As a result, the SM pattern for the suppression of FCNCs is automatically fulfilled, and the constraints on the scale of new physics from rare processes do not exclude the possibility of flavored degrees of freedom (the squarks) around or even slightly below 1 TeV.

In most flavour observables the MFV hypothesis implies small ( $\lesssim 10\%$ ) deviations from the SM (reason

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why this scenario is still consistent with present data). However, as I will discuss in the rest of this talk, a notable exception is provided by helicity-suppressed observables in the large  $\tan\beta$  regime of the MSSM. Here deviations from the SM at low energies can be large also under the MFV hypothesis, and improved data on low-energy observables turns out to be a key ingredient to identify this scenario.

## 2 MFV at large $\tan\beta$ : general considerations

The Higgs sector of the MSSM consists of two  $SU(2)_L$  scalar doublets, coupled separately to up- and down-type quarks

$$\mathcal{L}_Y^{\text{tree}} = \bar{Q}_L Y_U U_R H_U + \bar{Q}_L Y_D D_R H_D + \bar{\tilde{L}}_L Y_E E_R H_D + V(H_U, H_D) + \text{h.c.} \quad (3)$$

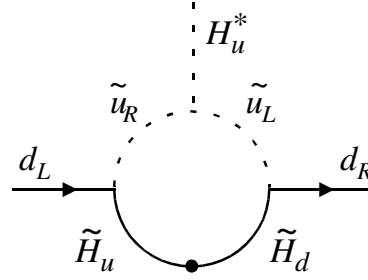
A key parameter of this sector is the ratio of the two Higgs vevs:  $\tan\beta = \langle H_U \rangle / \langle H_D \rangle$ . Varying  $\tan\beta$  leads to modify the overall normalization of the two Yukawa couplings (without changing their misalignment in flavour space). It is therefore not surprising that this parameter plays a key role in flavour physics, especially under the MFV hypothesis.

On the theoretical side, the large  $\tan\beta$  regime (or  $\tan\beta = \mathcal{O}(m_t/m_b) \approx 30 - 50$ ) has an intrinsic interest since it allows the unification of top and bottom Yukawa couplings, as predicted for instance in  $SO(10)$  models of grand unification (see e.g. Ref. [2]). As recently stressed in Ref. [3], an independent motivation of large  $\tan\beta$  is found in gauge-mediated models. Here the hierarchy  $\langle H_U \rangle \gg \langle H_D \rangle$  naturally follows from the tree-level conditions on the soft supersymmetry-breaking terms and the RGE evolution. In models with gauge-mediated supersymmetry-breaking the flavour structure of the model is also naturally consistent with the MFV hypothesis. As a result, a MFV scenario with large  $\tan\beta$  is not a construction *ad hoc* to analyse interesting effects in flavour physics: these two conditions can both be realized in well-motivated supersymmetry-breaking scenarios.

Before discussing the implications of the large  $\tan\beta$  regime in flavour-violating observables, it is worth discussing some general aspects of this scenario as well as its phenomenological motivations beside flavour physics.

### 2.1 The effective Yukawa interaction at large $\tan\beta$

The tree-level Lagrangian in Eq. (3) has an additional global symmetry with respect to the SM Yukawa interaction: it is invariant under an Abelian phase rotation of  $H_D$  and  $D_R$  fields with opposite charge ( $U(1)_{\text{PQ}}$  symmetry). However, this symmetry cannot be an exact symmetry of the full MSSM Lagrangian: it has to be broken at least in the scalar potential in order to avoid the presence of a massless pseudoscalar



**Fig. 1.** Typical non-holomorphic coupling of the  $H_U$  field to down-type quarks [4].

Higgs boson. Beyond the tree-level quantum corrections transmit the  $U(1)_{\text{PQ}}$  breaking to the Yukawa sector. As a result, the effective Yukawa interaction obtained by summing the leading quantum corrections may become substantially different from the one in Eq. (3).

The most interesting phenomenon induced by the  $U(1)_{\text{PQ}}$  breaking are effective non-holomorphic couplings of the  $H_U$  field to down-type quarks [4] (generated by one-loop diagrams of the type in Fig. 1 and similar diagrams with gluino exchange). Being generated only at the quantum level, these effective couplings are small ( $\epsilon \sim 1/(16\pi)^2$ ). However, since the  $H_U$  field has a large vev, the non-holomorphic terms induce large corrections to the vacuum structure of the theory in the large  $\tan\beta$  regime ( $\epsilon \tan\beta \sim 1$ ). Moreover, these one-loop amplitudes give rise to dimension-four effective operators, which are not suppressed in the limit of a heavy supersymmetry breaking scale.

Two steps are necessary to re-sum the leading non-decoupling corrections to all orders [1, 5]:

- the structure of the effective Yukawa interaction ( $\mathcal{L}_Y^{\text{eff}}$ ) must be determined starting from the one-loop effective potential, before determining the vacuum structure of the theory (i.e. independently of the spontaneous breaking of the  $SU(2)_L$  symmetry);
- the quark mass eigenstates must be defined by the diagonalization of the effective Yukawa interaction at the minimum of the Higgs potential.

By this way all the leading  $\mathcal{O}(\epsilon \tan\beta)$  terms are automatically included in the modified relations between Yukawa couplings and physical observables. The most notable consequences are the redefinition of the diagonal down-type Yukawa couplings in terms of quark masses [4]; a modification of the relation between CKM matrix elements and off-diagonal Yukawa couplings [4]; a modified structure for the charged-Higgs couplings to quarks [7, 8]. Last but not least, a sizable FCNC coupling of down-type quarks to the heavy neutral Higgs fields ( $H^0, A^0$ ) is generated [9, 5].<sup>1</sup>

<sup>1</sup> The construction of  $\mathcal{L}_Y^{\text{eff}}$  and the identification of the leading  $\mathcal{O}(\epsilon \tan\beta)$  terms is straightforward in the limit of the double hierarchy  $M_{\text{fermions}}^2 \gg M_H^2 \gg M_W^2$ , where we can neglect effective operators with dimension higher than

The latter phenomenon, which is particularly relevant for flavour physics, can easily be understood by noting that the diagram in Fig. 1 generates an effective interaction of the type

$$\delta\mathcal{L} = \epsilon_\chi \bar{Q}_L Y_U Y_U^\dagger Y_D D_R (H_U)^c. \quad (4)$$

This term is not aligned (in flavour space) and has a different composition of  $h^0$ ,  $H^0$ ,  $A^0$ , and vev components with respect to the leading term ( $\bar{Q}_L Y_D D_R H_D$ ). As a result, after the diagonalization of quark masses an effective FCNC coupling to the heavy fields  $H^0$  and  $A^0$  is generated. The strength of this FCNC is controlled by the off-diagonal entries of the CKM matrix, similarly to all the other FCNC amplitudes in a MFV framework. Being suppressed by the presence of  $Y_D$ , this effect is negligible in most FCNC amplitudes, but for the helicity-suppressed ones (where also the SM contribution is suppressed by down-type masses). In the latter case the impact of Higgs-mediated FCNC amplitudes can be quite in the large: the FCNC coupling grows quadratically with  $\tan\beta$  and is not suppressed in the limit of heavy squark masses.

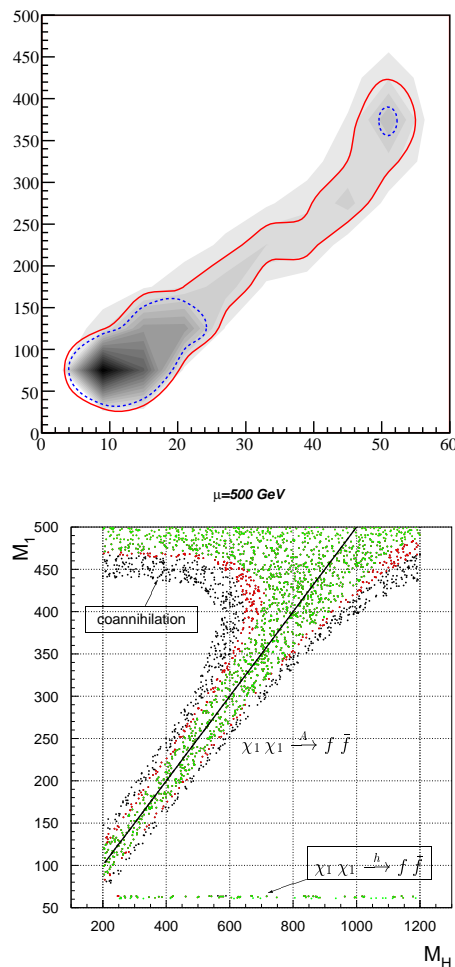
## 2.2 Phenomenological motivations in flavour-conserving processes

The MFV scenario with large  $\tan\beta$  is perfectly consistent with all the existing constraints from electroweak precision tests and flavour physics. An illustration of this statement is provided by the plot in Fig. 2, which is a projection in the  $M_H$ – $\tan\beta$  plane of a recent global fit [12] in the so-called constrained MSSM (CMSSM). The parameter space of the CMSSM is only a small subset of the general MSSM with MFV. As shown in Fig. 2, even within this restricted framework the corner with  $\tan\beta = \mathcal{O}(50)$  is well consistent with data and even slightly favored compared to other regions of the parameter space.

The shape of the plot in Fig. 2 is modeled by two flavour-conserving observables, which can be interpreted as the first two hints of low-energy supersymmetry: the anomalous magnetic of the muon and the neutralino relic abundance. As we will discuss below, in both cases large  $\tan\beta$  values are consistent and/or slightly favored by present data.

The possibility that the anomalous magnetic moment of the muon  $[a_\mu = (g-2)_\mu/2]$  provides a first hint of physics beyond the SM has been widely discussed in the recent literature [13]. As shown by Czarnecki [14], the consistency of the various  $e^+e^-$  experiments in the determination of  $(a_\mu)_{\text{had}}^{\text{SM}}$  has substantially increased our confidence in the SM prediction of this quantity. As a result, the discrepancy between the BNL measurement of  $a_\mu$  and its SM prediction is now a solid 3

six in  $\mathcal{L}_Y^{\text{eff}}$ . As discussed by Trine at this conference [10], this method can be extended also to the less trivial case  $M_H \sim M_W$ : this extension has allowed to clarify the controversial claim about new types of large  $\tan\beta$  effects in the  $M_H \sim M_W$  region [11].



**Fig. 2.** Up: Projection in the  $M_H$ – $\tan\beta$  plane of the global fit to the CMSSM [12]. Down: Allowed regions in the  $M_1$ – $M_H$  plane satisfying the relic density constraint  $\Omega h^2 < 0.119$  for  $M_{\tilde{q}} = 2M_{\tilde{l}} = |A_U| = 2\mu = 1$  TeV and  $\tan\beta = 20$  (inner points), 30 and 50 (all points) [18].

sigma effect:

$$\Delta a_\mu = a_\mu^{\text{exp}} - a_\mu^{\text{SM}} \approx (2.9 \pm 0.9) \times 10^{-9}. \quad (5)$$

The size of this discrepancy is about twice the electroweak SM contribution ( $\Delta a_\mu^{\text{e.w.}} \approx 1.5 \times 10^{-9}$ ). Given the great success of the SM in the electroweak sector, this fact is apparently very surprising. However, it can easily be explained noting that  $a_\mu$  is an helicity suppressed observable, whose non-standard contribution can be enhanced compared to the SM one by increasing the value of  $\tan\beta$ . Within the MSSM this enhancement can occur via gaugino-slepton loops, which generate a contribution to  $a_\mu$  proportional to the muon Yukawa coupling (and not to its mass) [15]. In the limit of almost degenerate soft-breaking terms, this can be written as

$$\Delta a_\mu^{\text{MSSM}} \approx \tan\beta \times \Delta a_\mu^{\text{e.w.}} \times \left( \frac{M_W}{\tilde{M}_{\text{slept}}} \right)^2. \quad (6)$$

For values of  $\tan\beta = \mathcal{O}(10)$  the  $M_W/\widetilde{M}_{\text{slept}}$  suppression can easily be compensated for sleptons well above the  $W$  mass, in perfect agreement with the constraints of electroweak precision tests.

Recent astrophysical observations consolidate the hypothesis that the universe is full of dark matter localized in large clusters [16]. The cosmological density of this type of matter, which is likely to be composed by stable and weakly-interactive massive particles, is determined with good accuracy

$$0.079 \leq \Omega_{\text{CDM}} h^2 \leq 0.119 \quad \text{at } 2\sigma \text{ C.L.} \quad (7)$$

A perfect candidate for such form of matter is the lightest neutralino of the MSSM (assuming  $R$ -parity conservation). In such case two key conditions need to be satisfied: i) the neutralino must be the lightest supersymmetric particle (LSP); ii) it must have a sufficiently large annihilation cross-section into SM matter.

The second condition, which is necessary given the large amount of neutralinos produced in the early universe compared the upper bound in Eq. (7), is not easily satisfied. In most of the phenomenologically-allowed regions of the MSSM the lightest neutralino (usually a  $B$ -ino) has a very low annihilation cross section. At low values of  $\tan\beta$  there are essentially two mechanisms which can enhance this cross-section: i) light sfermions (such that the  $t$ -channel sfermion exchange leads to a sufficiently large annihilation amplitude); ii) the co-annihilation with an almost degenerate NLSP.

The large  $\tan\beta$  region has the virtue of allowing a third enhancement mechanism for the annihilation cross-section of the relic neutralinos: the so-called  $A$  funnel region. Here the dominant neutralino annihilation amplitude is the  $s$ -channel heavy-Higgs exchange. As illustrated in Fig. 2 (down), the size of the allowed region for this mechanism grows with  $\tan\beta$  and it can become very large for  $\tan\beta = \mathcal{O}(50)$ . This is the main reason for the local maximum around  $\tan\beta \approx 50$  in Fig. 2 (up). As we will discuss in the following, Fulfilling the dark matter constraints via the  $A$ -funnel mechanism leads to well defined signatures in flavour physics. Indeed several of the parameters which control the amount of relic abundance also play a key role in flavour observables [18,19]

### 3 Large $\tan\beta$ effects in $B$ (and $K$ ) physics

In the MFV scenario we are considering the overall normalization of  $Y_D$  is largely enhanced compared to the SM case. However, its misalignment in flavour space with respect to  $Y_U$  is not modified. The latter property (following from the MFV ansatz) implies that flavour-changing observables not suppressed by powers of down-type quark masses (i.e. most of the experimentally accessible observables) are not sensitive to the value of  $\tan\beta$ . The interesting effects induced by  $\tan\beta \gg 1$  show up only in the few observables sensitive to helicity-suppressed amplitudes. These are

confined to the  $B$ -meson system (because of the large  $b$ -quark Yukawa coupling), with the notable exception of  $K \rightarrow \ell\nu$  decays. We can divide the most interesting observables in three classes: the charged-current processes  $B(K) \rightarrow \ell\nu$ , the rare decays  $B_{s,d} \rightarrow \ell^+\ell^-$ , and the FCNC transition  $B \rightarrow X_s\gamma$ .

#### 3.1 $B(K) \rightarrow \ell\nu$

The charged-current processes  $P \rightarrow \ell\nu$  are the simplest case. Here both SM and Higgs-mediated contributions (sensitive to  $\tan\beta$ ) are dominated by a tree-level amplitude. The SM branching ratio can be written as

$$\mathcal{B}(P \rightarrow \ell\nu)^{\text{SM}} = \frac{G_F^2 m_P m_\ell^2}{8\pi} \times \left(1 - \frac{m_\ell^2}{m_P^2}\right)^2 f_P^2 |V_{uq}|^2 \tau_P (1 + \delta_{\text{e.m.}}) \quad (8)$$

where  $V_{uq} = V_{ub}(V_{us})$  for  $P = B(K)$  and  $\delta_{\text{e.m.}}$  denotes the electromagnetic corrections.

Within two-Higgs doublet models, the  $H^\pm$ -exchange amplitude induces an additional tree-level contribution to semileptonic decays proportional to the Yukawa couplings of quarks and leptons [20]. This can compete with the  $W^\pm$  exchange only in  $P \rightarrow \ell\nu$  decays, thanks to the helicity suppression of the SM amplitude. Taking into account the resummation of the leading  $\tan\beta$  corrections to all orders, the  $H^\pm$  contributions to the  $P \rightarrow \ell\nu$  amplitude within a MFV supersymmetric framework leads to the following ratio [21,22]:

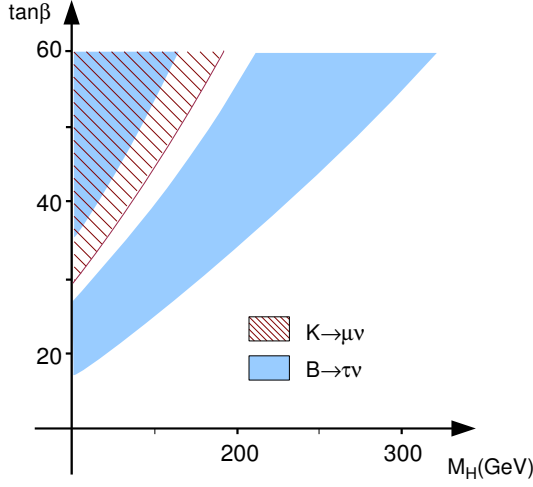
$$R_{P\ell\nu} = \frac{\mathcal{B}(P\ell\nu)}{\mathcal{B}^{\text{SM}}(P\ell\nu)} \stackrel{\text{SUSY}}{=} \left[1 - \left(\frac{m_P^2}{m_{H^\pm}^2}\right) \frac{\tan^2\beta}{(1 + \epsilon_0 \tan\beta)}\right]^2, \quad (9)$$

where  $\epsilon_0$  denotes the effective coupling which parametrizes the non-holomorphic corrections to the down-type Yukawa interaction [9,5]. For a natural choice of the MSSM parameters Eq. (9) implies a suppression with respect to the SM in  $B$  decays of few  $\times 10\%$  (but an enhancement is also possible for very light  $M_{H^\pm}$ ) and an effect 100 times smaller in  $K$  decays (where the branching ratio is necessarily smaller than  $\mathcal{B}^{\text{SM}}$ ).

In the  $B$  case only the  $\tau$  modes has been observed. The average of the latest results by Babar and Belle yields [23]  $\mathcal{B}(B \rightarrow \tau\nu)^{\text{exp}} = (1.41 \pm 0.43) \times 10^{-4}$ . In the Kaon system both decay modes ( $\ell = \mu, \nu$ ) are measured and the precision of  $\mathcal{B}(K \rightarrow \mu\nu)$  is around 0.3% [24]. Interestingly, the level of experimental precision in the combinations

$$\frac{1}{m_B^2} [R_{B \rightarrow \tau\nu} - 1] \quad \text{and} \quad \frac{1}{m_K^2} [R_{K \rightarrow \tau\nu}] \quad (10)$$

is comparable. In the limit of negligible theoretical errors we should therefore expect similar bounds in the



**Fig. 3.** Present constraints in the  $M_H$ - $\tan\beta$  plane from  $\mathcal{B}(B \rightarrow \tau\nu)$  and  $\mathcal{B}(K \rightarrow \mu\nu)$  [25].

$M_H$ - $\tan\beta$  plane from  $B$  and  $K$  decays. This limit is far from being realistic, due to the sizable errors on  $f_P$  (determined from Lattice QCD) and  $V_{uq}$  (which must be determined without using the information on  $P \rightarrow \ell\nu$  decays). But again the present level of precision is such that the  $B$  and  $K$  decays set competitive bounds in the  $M_H$ - $\tan\beta$  plane (see Fig. 3).

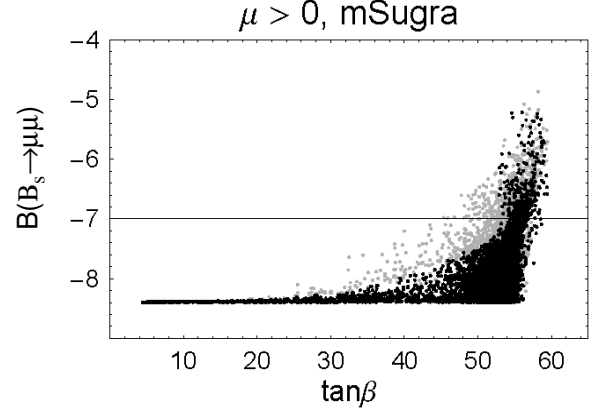
### 3.2 $B \rightarrow \ell^+\ell^-$

The important role of  $\mathcal{B}(B_{s,d} \rightarrow \ell^+\ell^-)$  in the large  $\tan\beta$  regime of the MSSM has been widely discussed in the literature (see e.g. Ref. [19, 21, 26, 27] for a recent discussion). Similarly to  $P \rightarrow \ell\nu$  decays, the leading non-SM contribution in  $B \rightarrow \ell^+\ell^-$  decays is generated by a single tree-level type amplitude: the neutral Higgs exchange  $B \rightarrow A, H \rightarrow \ell^+\ell^-$ . Since the effective FCNC coupling of the neutral Higgs bosons appears only at the quantum level, in this case the amplitude has a strong dependence on other MSSM parameters in addition to  $M_H$  and  $\tan\beta$ . In particular, a key role is played by  $\mu$  and the up-type trilinear soft-breaking term ( $A_U$ ), which control the strength of the diagram in Fig. 1. The leading parametric dependence of the scalar FCNC amplitude from these parameters is given by

$$\mathcal{A}(B \rightarrow A, H \rightarrow \ell^+\ell^-) \propto \frac{m_b m_\ell}{M_A^2} \frac{\mu A_U}{M_{\tilde{q}}^2} \tan^3 \beta \times f_{\text{loop}} \quad (11)$$

For  $\tan\beta \sim 50$  and  $M_A \sim 0.5$  TeV the neutral-Higgs contribution to  $\mathcal{B}(B_{s,d} \rightarrow \ell^+\ell^-)$  can easily lead to an  $\mathcal{O}(100)$  enhancement over the SM expectation. This possibility is already excluded by experiments: the upper bound  $\mathcal{B}(B_s \rightarrow \mu^+\mu^-) < 5.8 \times 10^{-8}$  [28] is only about 15 times higher than the SM prediction [29]

$$\mathcal{B}^{\text{SM}}(B_s \rightarrow \mu^+\mu^-) = (3.4 \pm 0.5) \times 10^{-9}. \quad (12)$$



**Fig. 4.**  $\mathcal{B}(B_s \rightarrow \mu^+\mu^-)$  as a function of  $\tan\beta$  in the mSUGRA scenario [19].

This limit poses interesting constraints on the MSSM parameter space, especially for light  $M_H$  and large values of  $\tan\beta$  (see e.g. Fig. 4). However, given the specific dependence on  $A_U$  and  $\mu$ , the present  $\mathcal{B}(B_s \rightarrow \mu^+\mu^-)$  bound does not exclude the large  $\tan\beta$  effects in  $(g-2)_\mu$  and  $P \rightarrow \ell\nu$  already discussed. The only clear phenomenological conclusion which can be drawn for the present (improved) limit on  $\mathcal{B}(B_s \rightarrow \mu^+\mu^-)$  is the fact that the neutral-Higgs contribution to  $\Delta M_{B_s}$  [30] is negligible.

### 3.3 $B \rightarrow X_s \gamma$

The last flavour physics observable we need to consider is  $\mathcal{B}(B \rightarrow X_s \gamma)$ . As is well known, this FCNC transition is particularly sensitive to non-standard contributions, not only in the large  $\tan\beta$  regime of the MSSM. Contrary to pure leptonic decays discussed before,  $B \rightarrow X_s \gamma$  does not receive effective tree-level contributions from the Higgs sector. The one-loop charged-Higgs amplitude, which increases the rate compared to the SM expectation, can be partially compensated by the chargino-squark amplitude, giving rise to delicate cancellations. As a result, the extraction of bound in the  $M_H$ - $\tan\beta$  plane from  $\mathcal{B}(B \rightarrow X_s \gamma)$  (within the MSSM) is a non trivial task.

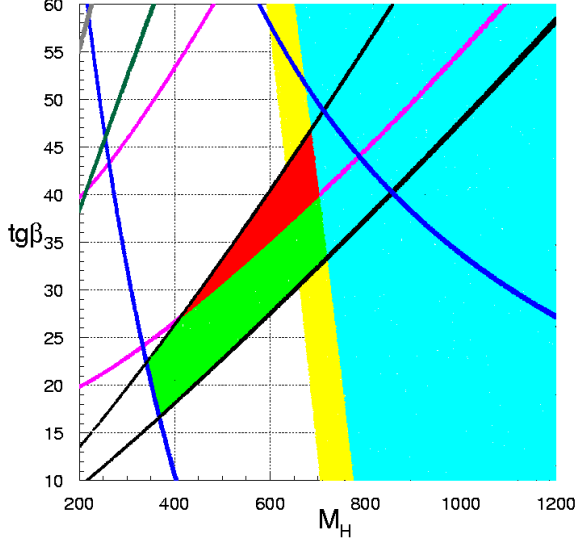
Despite the complicated interplay of various non-standard contributions,  $B \rightarrow X_s \gamma$  is particularly interesting given the good theoretical control of the SM prediction and the small experimental error. According to the recent NNLO analysis of Ref. [31], the SM prediction is

$$\mathcal{B}(B \rightarrow X_s \gamma)_{E_\gamma > 1.6 \text{ GeV}}^{\text{SM}} = (3.15 \pm 0.23) \times 10^{-4}, \quad (13)$$

to be compared with the experimental average [32]:

$$\mathcal{B}(B \rightarrow X_s \gamma)_{E_\gamma > 1.6 \text{ GeV}}^{\text{exp}} = (3.55 \pm 0.24) \times 10^{-4}. \quad (14)$$

These results allow a small but non negligible positive non-standard contribution to  $\mathcal{B}(B \rightarrow X_s \gamma)$  (as



**Fig. 5.** Combined bounds from low-energy observables in the  $\tan\beta$ – $M_H$  plane assuming heavy squarks and dark-matter constraints in the  $A$ -funnel region [21]. Main free parameters:  $M_{\tilde{q}} = 1.5$  TeV,  $A_U = -1$  TeV,  $\mu = 0.5$  TeV and  $M_{\tilde{t}} = 0.4$  TeV. Main low-energy constraints:  $1.01 < R_{B_s\gamma} < 1.24$  (region between the dark-gray (blue) lines falling at large  $M_H$ );  $2 < 10^9(a_\mu^{\text{exp}} - a_\mu^{\text{SM}}) < 4$  (region between the two gray (purple) lines raising at large  $M_H$ );  $0.8 < R_{B\tau\nu} < 0.9$  (region between the two black lines raising at large  $M_H$ ). The light-gray (light-blue) area is excluded by the dark-matter conditions.

expected if the charged-Higgs amplitude would dominate over the chargino-squark one), which represents one of the most significant constraint in the MSSM parameter space.

### 3.4 Discussion

The combined constraints on the three flavour observables discussed above leads to identify well-defined regions of the MSSM parameter space. For instance in the CMSSM  $\mathcal{B}(B \rightarrow X_s\gamma)$  plays a key role, together with  $(g-2)_\mu$ , in defining the preferred region showed in Fig. 2. As discussed in Ref. [33], in the NUHM scenario, where the universality condition between Higgs and sfermion soft breaking term is relaxed,  $\mathcal{B}(B \rightarrow \tau\nu)$  and  $\mathcal{B}(B \rightarrow \mu^+\mu^-)$  are the most significant constraints in the light  $M_H$  and large  $\tan\beta$  region.

There also well-motivated scenarios where the present constraints on this observables rule out most of the available parameter space. As shown in Ref. [34], the present bound from  $\mathcal{B}(B \rightarrow \tau\nu)$  puts in serious difficulties the SO(10) GUT model of Dermisek and Raby [35], which is a specific example of MFV scenario with large  $\tan\beta$ .

An illustration of the typical correlations of the low-energy constraints in the  $M_H$ – $\tan\beta$ , in a generic

scenario with heavy squarks and dark-matter conditions satisfied in the  $A$ -funnel region, is shown in Fig. 5. The assumption of heavy squarks leads to a substantial simplification in the description of the large  $\tan\beta$  effects, and in this regime we can draw the following general conclusions [18]: 1) The  $B \rightarrow X_s\gamma$  constraint is always easily satisfied for  $M_H \gtrsim 300$  GeV, or even lighter  $M_H$  for large  $\tan\beta$  values. This is because the present experimental range allows a significant (positive) non-standard contribution to the  $B \rightarrow X_s\gamma$  rate, and choosing  $A_U < 0$  the positive charged-Higgs contribution is partially compensated by the negative chargino-squarks amplitude. 2) The present limit on  $B \rightarrow \mu^+\mu^-$  is not particularly stringent. 3) A supersymmetric contribution to  $a_\mu$  of  $\mathcal{O}(10^{-9})$  is perfectly compatible with the present constraints from  $\mathcal{B}(B \rightarrow X_s\gamma)$ , especially for  $A_U < 0$ . Taking into account the correlation between neutralino and charged-Higgs masses occurring in the  $A$ -funnel region, this implies a suppression of  $\mathcal{B}(B \rightarrow \tau\nu)$  with respect to its SM prediction of at least 10%. A more precise determination of  $\mathcal{B}(B \rightarrow \tau\nu)$  is therefore a key element to test this scenario.

## 4 Lepton Flavour Violation and LF non-universality

LFV couplings naturally appear in the MSSM once we extend it to accommodate the non-vanishing neutrino masses and mixing angles by means of a supersymmetric seesaw mechanism [36]. In particular, the renormalization-group-induced LFV entries appearing in the left-handed slepton mass matrices have the following form [36]:

$$\delta_{LL}^{ij} = \frac{(M_\ell^2)_{L_i L_j}}{\sqrt{(M_\ell^2)_{L_i L_i} (M_\ell^2)_{L_j L_j}}} = c_\nu (Y_\nu^\dagger Y_\nu)_{ij}, \quad (15)$$

where  $Y_\nu$  are the neutrino Yukawa couplings and  $c_\nu$  is a numerical coefficient, depending on the SUSY spectrum, typically of  $\mathcal{O}(0.1-1)$ . As is well known, the information from neutrino masses is not sufficient to determine in a model-independent way all the seesaw parameters relevant to LFV rates and, in particular, the neutrino Yukawa couplings. To reduce the number of free parameters specific SUSY-GUT models and/or flavour symmetries need to be employed. Two main roads are often considered in the literature: the case where the charged-lepton LFV couplings are linked to the CKM matrix (the quark mixing matrix) and the case where they are connected to the PMNS matrix (the neutrino mixing matrix) [37]. These two possibilities can be formulated in terms of well-defined flavour-symmetry structures starting from the MFV hypothesis [38, 39].

Once non-vanishing LFV entries in the slepton mass matrices are generated, LFV rare decays are naturally induced by one-loop diagrams with the exchange of



gauginos and sleptons. For large values of  $\tan\beta$  the radiative decays  $\ell_i \rightarrow \ell_j \gamma$ , mediated by dipole operators, are linearly enhanced, in close analogy to the  $\tan\beta$ -enhancement of  $\Delta a_\mu = (g_\mu - g_\mu^{\text{SM}})/2$ . A strong link between these two observable naturally emerges [40]. We can indeed write

$$\frac{\mathcal{B}(\ell_i \rightarrow \ell_j \gamma)}{\mathcal{B}(\ell_i \rightarrow \ell_j \nu_{\ell_i} \nu_{\ell_j})} = \frac{48\pi^3 \alpha}{G_F^2} \left[ \frac{\Delta a_\mu}{m_\mu^2} \right]^2 \times \left[ \frac{f_{2c}(M_2^2/M_\ell^2, \mu^2/M_\ell^2)}{g_{2c}(M_2^2/M_\ell^2, \mu^2/M_\ell^2)} \right]^2 |\delta_{LL}^{ij}|^2, \quad (16)$$

where  $f_{2c}$  and  $g_{2c}$  are  $\mathcal{O}(1)$  loop functions. In the limit of a degenerate SUSY spectrum, this implies

$$\mathcal{B}(\ell_i \rightarrow \ell_j \gamma) \approx \left[ \frac{\Delta a_\mu}{20 \times 10^{-10}} \right]^2 \times \begin{cases} 1 \times 10^{-4} |\delta_{LL}^{12}|^2 & [\mu \rightarrow e] \\ 2 \times 10^{-5} |\delta_{LL}^{23}|^2 & [\tau \rightarrow \mu] \end{cases}. \quad (17)$$

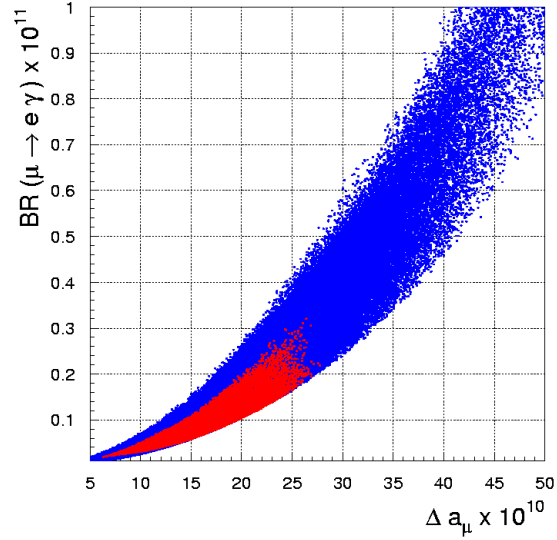
The strong correlation between  $\Delta a_\mu$  and the rate of the LFV transitions  $\ell_i \rightarrow \ell_j \gamma$  holds well beyond the simplified assumptions used to derive these equations (see Fig. 6). The normalization  $|\delta_{LL}^{12}| = 10^{-4}$  used in Fig. 6 for  $\mathcal{B}(\mu \rightarrow e \gamma)$  corresponds to the pessimistic MFV hypothesis in the lepton sector [38]. As can be seen, for such natural choice of  $\delta_{LL}$  the  $\mu \rightarrow e \gamma$  branching ratio is in the  $10^{-12}$  range, i.e. well within the reach of the MEG experiment [41].

An independent and potentially large class of LFV contributions to rare decays in the large  $\tan\beta$  regime of the MSSM comes from Higgs-mediated amplitudes. Similarly to the quark sector, non-holomorphic couplings can induce an effective FCNC Higgs coupling also in the lepton sector [42]. Gauge- and Higgs-mediated amplitudes leads to very different correlations among LFV processes [37, 43, 44] and that their combined study can reveal the underlying mechanism of LFV.

Finally, as recently pointed out in Ref. [45], Higgs-mediated LFV effects at large  $\tan\beta$  can also induce visible deviations of lepton-flavour universality in charged-current processes. If the slepton sector contains sizable (non-minimal) sources of LFV, we could hope to observe deviations from the SM predictions in the  $\mathcal{B}(P \rightarrow \ell \nu)/\mathcal{B}(P \rightarrow \ell' \nu)$  ratios. The deviations can be  $\mathcal{O}(1\%)$  in  $\mathcal{B}(K \rightarrow e \nu)/\mathcal{B}(K \rightarrow \mu \nu)$  [45], and can reach  $\mathcal{O}(1)$  and  $\mathcal{O}(10^3)$  in  $\mathcal{B}(B \rightarrow \mu \nu)/\mathcal{B}(B \rightarrow \tau \nu)$  and  $\mathcal{B}(B \rightarrow e \nu)/\mathcal{B}(B \rightarrow \tau \nu)$ , respectively [21].

## 5 Conclusions

Within the Minimal Supersymmetric extension of the Standard Model, the scenario with large  $\tan\beta$  and Minimal Flavour Violation is well motivated and phenomenologically allowed. In this framework one could naturally accommodate the present (non-standard) central value of  $(g-2)_\mu$ , explain why the lightest Higgs



**Fig. 6.**  $\mathcal{B}(\mu \rightarrow e \gamma)$  vs.  $\Delta a_\mu = (g_\mu - g_\mu^{\text{SM}})/2$ , assuming  $|\delta_{LL}^{12}| = 10^{-4}$ . The scatter plot has been obtained employing the following ranges:  $300 \text{ GeV} \leq M_{\tilde{t}} \leq 600 \text{ GeV}$ ,  $200 \text{ GeV} \leq M_2 \leq 1000 \text{ GeV}$ ,  $500 \text{ GeV} \leq \mu \leq 1000 \text{ GeV}$ ,  $10 \leq \tan\beta \leq 50$ , and setting  $A_U = -1 \text{ TeV}$ ,  $M_{\tilde{q}} = 1.5 \text{ TeV}$ . Moreover, the GUT relations  $M_2 \approx 2M_1$  and  $M_3 \approx 6M_1$  are assumed. The internal (red) area correspond to points within the  $A$ -funnel region [21]

boson has not been observed yet, and why no signal of new physics has been observed yet in  $\mathcal{B}(B \rightarrow X_s \gamma)$  and other flavour physics observables. Moreover, spectacular deviations from the SM in low-energy processes such as  $B \rightarrow \mu^+ \mu^-$  or  $\mu \rightarrow e \gamma$  could be just around the corner.

One of the interesting aspects of this scenario is the strong interplay between low-energy physics and direct new-physics searches at high energy. As I tried to outline in this talk, improved measurements in the flavour sector, particularly in the helicity suppressed decays  $B(K) \rightarrow \ell \nu$ ,  $B \rightarrow X_s \gamma$ , and  $B \rightarrow \mu^+ \mu^-$  represent a very useful tool to restrict the parameter space of the model, even in absence of sizable deviations from the SM.

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## References

1. G. D'Ambrosio, G. F. Giudice, G. Isidori and A. Strumia, Nucl. Phys. B **645**, 155 (2002) [hep-ph/0207036].
2. G. Anderson, S. Raby, S. Dimopoulos, L. J. Hall and G. D. Starkman, Phys. Rev. D **49** (1994) 3660 [hep-ph/9308333]; T. Blazek, R. Dermisek and S. Raby, Phys. Rev. D **65** (2002) 115004 [hep-ph/0201081].

3. J. Hisano and Y. Shimizu, arXiv:0706.3145 [hep-ph]; see also M. Dine, Y. Nir and Y. Shirman, Phys. Rev. D **55** (1997) 1501 [arXiv:hep-ph/9607397]; R. Rattazzi and U. Sarid, Nucl. Phys. B **501** (1997) 297 [arXiv:hep-ph/9612464].
4. L. J. Hall, R. Rattazzi and U. Sarid, Phys. Rev. D **50** (1994) 7048 [arXiv:hep-ph/9306309].
5. G. Isidori and A. Retico, JHEP **0111** (2001) 001 [arXiv:hep-ph/0110121].
6. T. Blazek, S. Raby and S. Pokorski, Phys. Rev. D **52** (1995) 4151 [arXiv:hep-ph/9504364].
7. M. Carena, D. Garcia, U. Nierste and C. E. M. Wagner, Nucl. Phys. B **577**, 88 (2000) [hep-ph/9912516].
8. G. Degrandi, P. Gambino and G. F. Giudice, JHEP **0012** (2000) 009 [hep-ph/0009337]; M. Carena, D. Garcia, U. Nierste and C. E. Wagner, Phys. Lett. B **B499** (2001) 141 [hep-ph/0010003].
9. C. Hamzaoui, M. Pospelov and M. Toharia; Phys. Rev. D **59** (1999) 095005 [hep-ph/9807350]. K. S. Babu and C. F. Kolda, Phys. Rev. Lett. **84** (2000) 228 [arXiv:hep-ph/9909476].
10. S. Trine, these proceedings.
11. A. Freitas, E. Gasser and U. Haisch, Phys. Rev. D **76** (2007) 014016 [arXiv:hep-ph/0702267].
12. O. Buchmueller *et al.*, arXiv:0707.3447 [hep-ph].
13. See e.g. D. W. Hertzog *et al.* arXiv:0705.4617 [hep-ph]; F. Jegerlehner, arXiv:hep-ph/0703125.
14. A. Czarnecki, these proceedings.
15. See S. P. Martin and J. D. Wells, Phys. Rev. D **64** (2001) 035003 [hep-ph/0103067]; T. Moroi, Phys. Rev. D **53** (1996) 6565 [Erratum-ibid. D **56** (1997) 4424] [arXiv:hep-ph/9512396] and references therein.
16. D. N. Spergel *et al.* [WMAP Collaboration], Astrophys. J. Suppl. **170**, 377 (2007) [arXiv:astro-ph/0603449].
17. J. R. Ellis, K. A. Olive, Y. Santoso and V. C. Spanos, Phys. Lett. B **565** (2003) 176 [hep-ph/0303043]; G. Bertone, D. Hooper and J. Silk, Phys. Rept. **405** (2005) 279 [hep-ph/0404175], S. Profumo and C. E. Yaguna, Phys. Rev. D **70** (2004) 095004 [hep-ph/0407036].
18. G. Isidori, F. Mescia, P. Paradisi and D. Temes, Phys. Rev. D **75** (2007) 115019.
19. E. Lunghi, W. Porod and O. Vives, Phys. Rev. D **74** (2006) 075003 [arXiv:hep-ph/0605177].
20. W. S. Hou, Phys. Rev. D **48** (1993) 2342.
21. G. Isidori and P. Paradisi, Phys. Lett. B **639** (2006) 499 [hep-ph/0605012].
22. A. G. Akeroyd and S. Recksiegel, J. Phys. G **29**, 2311 (2003) [hep-ph/0306037].
23. D. Monorchio, talk presented at the 2007 Europhysics Conference on High Energy Physics (19-25 July 2007, Manchester, England), <http://www.hep.man.ac.uk/HEP2007/>
24. F. Ambrosino *et al.* [KLOE Collaboration], Phys. Lett. B **632** (2006) 76 [arXiv:hep-ex/0509045].
25. Flavianet Kaon WG, <http://www.lnf.infn.it/wg/vus/>
26. M. S. Carena, A. Menon, R. Noriega-Papaqui, A. Szykman and C. E. M. Wagner, Phys. Rev. D **74** (2006) 015009 [arXiv:hep-ph/0603106].
27. J. R. Ellis, S. Heinemeyer, K. A. Olive, A. M. Weber and G. Weiglein, JHEP **0708** (2007) 083 [arXiv:0706.0652 [hep-ph]].
28. A. Maciel, talk presented at the 2007 Europhysics Conference on High Energy Physics (19-25 July 2007, Manchester, England), <http://www.hep.man.ac.uk/HEP2007/>
29. A. J. Buras, Phys. Lett. B **566** (2003) 115 [arXiv:hep-ph/0303060].
30. A. J. Buras, P. H. Chankowski, J. Rosiek and L. Slawianowska, Nucl. Phys. B **659** (2003) 3 [hep-ph/0210145]; Phys. Lett. B **546** (2002) 96 [hep-ph/0207241]; Nucl. Phys. B **619** (2001) 434 [hep-ph/0107048].
31. M. Misiak *et al.*, hep-ph/0609232.
32. E. Barberio *et al.* [Heavy Flavor Averaging Group (HFAG)], hep-ex/0603003.
33. J. R. Ellis, S. Heinemeyer, K. A. Olive and G. Weiglein, Phys. Lett. B **653** (2007) 292 [arXiv:0706.0977 [hep-ph]].
34. M. Albrecht, W. Altmannshofer, A. J. Buras, D. Guadagnoli and D. M. Straub, arXiv:0707.3954 [hep-ph].
35. R. Dermisek and S. Raby, Phys. Lett. B **622** (2005) 327 [arXiv:hep-ph/0507045].
36. F. Borzumati and A. Masiero, Phys. Rev. Lett. **57**, 961 (1986);
37. E. Arganda and M. J. Herrero, Phys. Rev. D **73** (2006) 055003 [arXiv:hep-ph/0510405]; S. Antusch, E. Arganda, M. J. Herrero and A. M. Teixeira, JHEP **0611** (2006) 090 [arXiv:hep-ph/0607263].
38. V. Cirigliano, B. Grinstein, G. Isidori and M. B. Wise, Nucl. Phys. B **728**, 121 (2005) [hep-ph/0507001].
39. B. Grinstein, V. Cirigliano, G. Isidori and M. B. Wise, Nucl. Phys. B **763** (2007) 35 [hep-ph/0608123].
40. J. Hisano, T. Moroi, K. Tobe, M. Yamaguchi and T. Yanagida, Phys. Lett. B **357** (1995) 579 [hep-ph/9501407].
41. M. Grassi [MEG Collaboration], Nucl. Phys. Proc. Suppl. **149** (2005) 369.
42. K. S. Babu and C. Kolda, Phys. Rev. Lett. **89** (2002) 241802 [arXiv:hep-ph/0206310].
43. P. Paradisi, JHEP **0602**, 050 (2006) [arXiv:hep-ph/0508054];
44. P. Paradisi, JHEP **0608**, 047 (2006) [arXiv:hep-ph/0601100].
45. A. Masiero, P. Paradisi and R. Petronzio, Phys. Rev. D **74**, 011701 (2006) [arXiv:hep-ph/0511289].